EXERCISES.

236

IF p, q, r, s are the lengths, supposed unequal, of the sides of a quadrilateral, prove that

$$[(p+q)(p+r)(p+s)(q+r)(q+s)(r+s)]^2 > [(p+q+r-s)(p+q-r+s)(p-q+r+s)(-p+q+r+s)]^3.$$

[*R. H. Graves.*]

237

THE axes of an ellipse are given, and one focal distance of a point on the curve. Find the ordinate of the point drawn to the major axis.

[*O. L. Mathiot.*]

238

FROM the point P on an equilateral hyperbola the ordinate and the supplemental chords are drawn. Find the locus of the join of the ordinate and the perpendicular bisector of the longer chord. [*R. H. Graves.*]

239

FIND the equation to QR of Exercise 205, P being given.

[*R. H. Graves.*]

240

FIND the centre locus of the circumconic

$$u\beta\gamma + v\gamma\alpha + w\alpha\beta = 0,$$

when uvw describes the straight line

$$l\alpha + m\beta + n\gamma = 0,$$

and determine the nature and position of the locus;

1. When the line is at infinity,

2. When the line is the axis of homology of the triangle of reference and its pedal triangle. [*R. H. Graves.*]

241

FIND the eccentricity of the ellipse which cuts a confocal lemniscate where its ordinate is a maximum, and the angle of intersection. [*R. H. Graves.*]

242

THE curve $\tan x + \tan y = a$ is symmetrical with regard to certain lines parallel to $x + y = 0$. [*Frank Morley.*]

243

A NODAL quartic passes through the twelve intersections of three conics. Show that the polars of the node with regard to the conics meet in a point. [*Frank Morley.*]

244

THE angle of intersection of a meridian of the earth with a confocal oval of Cassini equals the reduction of latitude at the join. [*R. H. Graves.*]

245

THE probability that an event A happens is p_1; the probability that an event B happens is p_2; the probability that neither happens is p_3; required, the probability that both happen. [*L. M. Hoskins.*]

246

FIND the average distance of a given point in the surface of a circle from the circumference. [*Artemas Martin.*]

247

Two sides of a triangle are a and b; find the average length of the third side. [*Artemas Martin.*]

248

A POINT is taken at random in a side of a square and a random line drawn from it to the opposite side. Find the average length of the random line. [*Artemas Martin.*]

249

CUT the two edges AB, CD of the tetraedron $ABCD$ by plane Π in P, Q respectively; and take P', Q' the harmonic conjugates to PQ relative to AB, CD. Draw a plane Σ through $P'Q'$ and let M, N be the points in which it cuts AC, BD. Then will the join MN intersect both PQ and $P'Q'$; and in points R, R' which divide MN harmonically. [*E. H. Moore, Jr.*]

250

A HOMOGENEOUS sphere rests on another such sphere of equal mass, which rests on a table. Everything being smooth and the system being slightly shaken, show that the spheres will separate when the upper one has turned through the angle $\cos^{-1}(\sqrt{3} - 1)$. [*Frank Morley.*]